Construction of space distribution of spatial density for catalogued objects

Physical meaning of spatial density is the number of objects in volume unit (1/km³). We use a deterministic approach. The easiest way to calculate space distribution of objects sizing larger than 10-30 cm (cataloged) is based on the catalogue data and the corresponding motion model. Naturally, this path is possible only if we have a full catalogue which in the form of TLE can download online at http://www.space-track.org.

This method includes the following basic operations:
a) the downloading the catalogue data from Internet and saving ones in the file "TLE_cat.txt";
b) the step-by-step reading of orbital parameters from the file;
c) the recalculation of TLE into altitude and latitude of objects at various time instants using the analytical motion model;
d) the summarize the number of cases of falling of objects into 2D-boxes of space partition with altitude step of 100 km and latitude step of 5 degree;
e) the statistical averaging of obtained data about number of objects in each box and the calculation of spatial density.

Execution of mentioned calculations for various time instants and averaging of results make possible to obtain enough sustainable estimates.
The algorithm parameters:

minimal altitude - 200 km;
maximal altitude – 2000 km;
number of prediction for each of TLE sets -100.

Part of catalogue data in form of TLE (example):

```
1 00005U 58002B 09077.87311246 .00000120 00000-0 12737-3 0 4211
2 00005 034.2562 275.8153 1850740 237.0392 103.8719 10.83996109759300
1 00011U 59001A 09079.04576453 .00000233 00000-0 11691-3 0 6947
2 00011 032.8671 320.3091 1483276 158.6056 208.3735 11.82483272129672
1 00012U 59001B 09078.89496355 .00000283 00000-0 16897-3 0 4612
2 00012 032.8952 290.3006 1680143 128.8394 247.4501 11.41219566 52468
. . . . . .
. . . . . .
1 34607U 98067BP 09079.09171105 .00776147 00000-0 52516-2 0 27
2 34607 051.6415 043.4237 0011670 135.4958 309.6759 15.72155431 22
1 34608U 98067BQ 09079.21946907 .00751243 00000-0 50025-2 0 25
2 34608 051.6390 042.7636 0012138 131.3654 317.6993 15.72598773 49
```

Data for each satellite consists of two lines in the following format:

```
1 NNNNNU NNNNNAAA NNNNN.NNNNNNNN +.NNNNNNNN +NNNNN-N +NNNNN-N N NNNNN
2 NNNNN NNN.NNNN NNN.NNNN NNNNNNN NNN.NNNN NNN.NNNN NN.NNNNNNNNNNNNN
```

Lines 1 and 2 are the standard Two-Line Orbital Element Set Format identical to that used by NORAD and NASA. The format description is:

Line 1

Column	Description
01	Line Number of Element Data
03-07	Satellite Number
08	Classification (U=Unclassified)
10-11	International Designator, last two digits of launch year, 2000+ if < 57.
12-14	International Designator, launch number of the year
15-17	International Designator, piece of the launch
19-20	Epoch Year, last two digits of year, 2000+ if < 57
21-32	Epoch Day of the year and fractional portion of the day
34-43	First Time Derivative of the Mean Motion
45-52	Second Time Derivative of Mean Motion (decimal point assumed)
54-61	BSTAR drag term (decimal point assumed)
63	Ephemeris type
65-68	Element number
69	Checksum (Modulo 10) (Letters, blanks, periods, plus signs = 0; minus signs = 1)

Line 2

Column	Description
01	Line Number of Element Data
03-07	Satellite Number
09-16	Inclination [Degrees]
18-25	Right Ascension of the Ascending Node [Degrees]
27-33	Eccentricity (decimal point assumed)
35-42	Argument of Perigee [Degrees]
44-51	Mean Anomaly [Degrees]
53-63	Mean Motion [Revs per day]
64-68	Revolution number at epoch [Revs]
69	Checksum (Modulo 10)

Recalculation of TLE into 6-dimensional position and velocity state vector is performed using SGP4 module, which includes a number of procedures:

- constants;
-initialization of variables;
-secular perturbations;
-long-periodic perturbations;

- calculation of argument of latitude;
-short-periodic perturbations;
- transformation matrix;
- calculation of 6-dimensional position and velocity state vector.

The values of the position vector allows us: a) to calculate the altitude and latitude of object, b) to place it in corresponding “box” and c) to repeat this procedures for each catalogue object. The statistical averaging of the results is achieved by iterate repetition of this procedures for random time instants. It should be noted that the interval of random time instants must not be very large, because otherwise some objects with low perigee or large ballistic coefficient may be reentered.

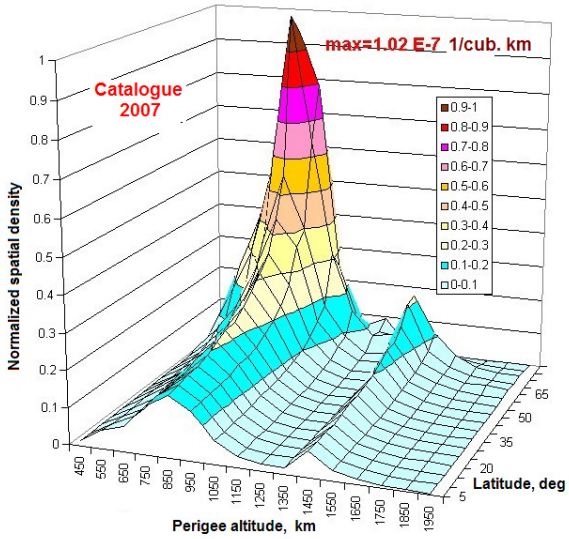

Figure 1. Spatial density versus altitude and latitude of point

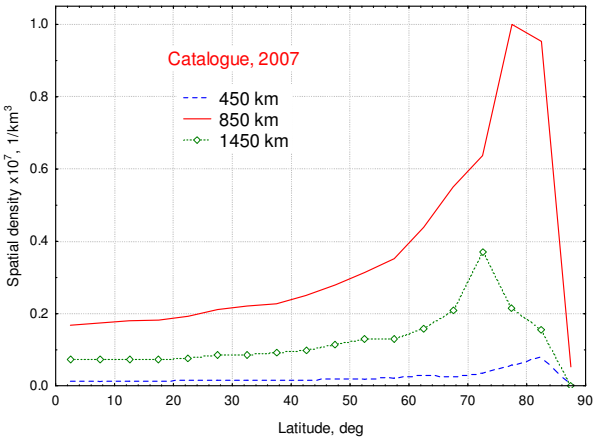

Figure 2. Spatial density versus latitude

Results (the example): It was performed the evaluation of spatial density of objects on base of catalogue for October 2007 in altitude range from 400 to 2000 km. The general view of the resulting distribution of spatial density versus altitude and latitude is shown in Figure 1. Two known maximum of density is clearly visible: a) the first global one equals 1.02 E-7 km-3, at altitudes of 800-900 km and latitude of 80-85

degrees and b) second local one equals 3.57 E-8 km-3, at an altitude of 1400-1500 km and latitude of 70-75 degrees. There has been sharp decrease of spatial density in area of 90 degree, as seen from data of figure 2 as well. Evaluation was carried out at 1000 realizations of random selection of time instants that is afforded a sufficient level of averaging.

It was preformed the study of accuracy of results as function of random selection number for various time instants. Relative errors were calculated as

$$\Delta = \left(\frac{1}{n} \sum_{ij} \frac{\left| \rho_{ij}^{(1)} - \rho_{ij}^{(2)} \right|}{\rho_{ij}^{(\max)}} \right) \cdot 100\%$$

Here:

$\rho_{ij}^{(1)}$ is the value of spatial density obtained with the specified number of realizations;

$\rho_{ij}^{(2)}$ is the value of spatial density obtained with 1000 realizations of random process;

$\rho_{ij}^{(\max)}$ is the absolute maximum of spatial density.

Dependence of relative errors versus number of realizations is presented in Figure 3.

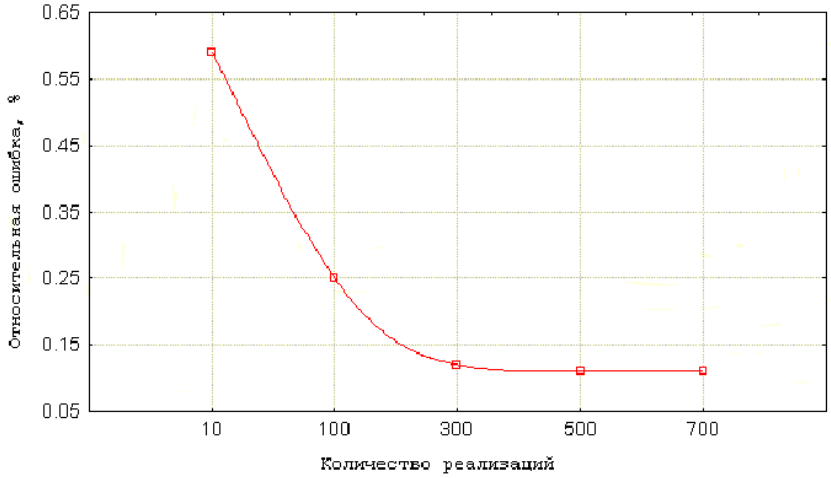

Figure 3. Δ values versus number of implementations

Average relative error for 100 realizations does not exceed 0.25%. Calculation time takes not more than 1-2 minutes on modern PC.

Output files:

No	File name	Comments
1	p_bh_cat.dat	Normalized spatial density versus altitude and latitude and the maximal value
2	p_h_cat.dat	Spatial density averaged by latitude
3	n_bh_cat.dat	Average number of objects in boxes

Example of file “p_bh_cat.dat” (for 2007).

250 0.004 0.004 0.003 0.004 0.004 0.003 0.003 0.004 0.004 0.003 0.004 0.004 0.005 0.004 0.004 0.005 0.006 0.000
350 0.014 0.012 0.013 0.013 0.014 0.014 0.015 0.018 0.018 0.020 0.021 0.022 0.034 0.015 0.018 0.020 0.022 0.000

450 0.013 0.012 0.013 0.013 0.015 0.015 0.015 0.016 0.016 0.018 0.018 0.021 0.029 0.026 0.035 0.058 0.079 0.004
550 0.050 0.046 0.044 0.046 0.050 0.053 0.052 0.048 0.053 0.053 0.060 0.071 0.092 0.099 0.114 0.174 0.237 0.015
650 0.069 0.073 0.073 0.080 0.087 0.092 0.091 0.088 0.091 0.100 0.110 0.130 0.168 0.183 0.233 0.302 0.389 0.020
750 0.132 0.135 0.133 0.136 0.140 0.153 0.153 0.151 0.175 0.180 0.197 0.234 0.290 0.361 0.477 0.544 0.614 0.168
850 0.168 0.174 0.180 0.182 0.193 0.211 0.221 0.227 0.250 0.279 0.314 0.352 0.439 0.550 0.637 1.000 0.953 0.053
950 0.133 0.136 0.137 0.142 0.144 0.155 0.156 0.163 0.180 0.198 0.223 0.270 0.363 0.396 0.403 0.642 0.795 0.054
1050 0.065 0.064 0.067 0.066 0.069 0.074 0.075 0.079 0.084 0.091 0.101 0.120 0.149 0.176 0.173 0.316 0.265 0.156
1150 0.030 0.030 0.033 0.032 0.032 0.037 0.037 0.039 0.039 0.046 0.052 0.059 0.080 0.090 0.098 0.150 0.094 0.068
1250 0.019 0.021 0.021 0.022 0.022 0.024 0.023 0.026 0.025 0.030 0.032 0.035 0.045 0.048 0.051 0.081 0.072 0.094
1350 0.018 0.020 0.020 0.020 0.023 0.023 0.024 0.025 0.028 0.030 0.034 0.040 0.048 0.053 0.076 0.092 0.063 0.022
1450 0.072 0.072 0.073 0.072 0.076 0.085 0.086 0.093 0.098 0.115 0.129 0.131 0.159 0.208 0.370 0.214 0.156 0.001
1550 0.028 0.028 0.030 0.029 0.030 0.033 0.031 0.033 0.037 0.043 0.047 0.052 0.061 0.075 0.127 0.108 0.019 0.002
1650 0.015 0.014 0.015 0.015 0.014 0.017 0.014 0.016 0.017 0.021 0.022 0.029 0.032 0.035 0.053 0.051 0.019 0.001
1750 0.007 0.006 0.006 0.006 0.007 0.007 0.007 0.008 0.009 0.009 0.011 0.013 0.015 0.017 0.024 0.021 0.009 0.001
1850 0.005 0.004 0.004 0.006 0.005 0.005 0.004 0.006 0.005 0.006 0.009 0.009 0.016 0.009 0.008 0.012 0.004 0.002
1950 0.004 0.004 0.003 0.004 0.004 0.004 0.005 0.004 0.004 0.005 0.006 0.007 0.012 0.007 0.004 0.010 0.003 0.001

Maximum of spatial density= 1.018E-0007

Example of file "p_h_cat.dat" (for 2007).

```
 250 2.2E-0010
 350 8.4E-0010
 450 1.8E-0009
 550 6.1E-0009
 650 1.1E-0008
 750 1.9E-0008
 850 2.8E-0008
 950 2.0E-0008
1050 9.4E-0009
1150 4.6E-0009
1250 2.9E-0009
1350 3.0E-0009
1450 1.1E-0008
1550 4.1E-0009
1650 2.0E-0009
1750 9.1E-0010
1850 6.3E-0010
1950 5.0E-0010
```

Example of file "n_bh_cat.dat" (for 2007).

```
 450   7   6   7   6   7   7   6   7   6   6   6   6   7   5   5   7   5   0 107
 550  27  24  23  24  25  25  23  21  21  19  20  20  23  20  18  20  17   0 370
 650  38  40  39  42  44  45  42  39  37  37  37  38  43  39  39  36  28   0 663
 750  75  76  74  74  74  77  73  68  73  69  68  71  76  78  81  67  45  41224
 850  98 101 103 101 104 109 109 105 108 110 111 110 118 123 112 126  73  11822
 950  80  81  80  81  80  83  79  77  80  80  82  87 100  91  73  83  62  11380
1050  40  39  40  39  39  40  39  38  38  38  38  40  43  42  32  42  21   4 653
1150  19  19  21  20  19  21  20  20  18  20  20  20  23  22  19  21   8   2 329
1250  12  13  13  14  14  14  13  14  12  13  13  12  13  12  10  11   6   3 212
1350  12  13  13  12  14  14  13  13  14  14  14  14  15  14  15  13   5   1 224
1450  49  49  48  47  48  52  50  50  49  53  54  48  50  54  76  32  14   0 825
1550  19  20  20  20  20  21  19  19  19  20  20  20  20  20  27  16   2   0 321
1650  10  10  10  10  10  11   9   9   9  10  10  11  11  10  11   8   2   0 161
1750   5   5   4   4   5   5   4   4   5   5   5   5   5   5   5   3   1   0  76
1850   3   3   3   4   4   3   3   3   3   3   4   4   6   3   2   2   0   0  54
1950   3   3   2   3   3   3   3   3   2   3   3   3   4   2   1   2   0   0  43
```

Left column shows the altitude values and right column shows total average number of objects within a given altitude layer.